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KINEMATIC AND DYNAMIC MODELS OF MANIPULATORS FOR USE IN SPACE: THE CONCEPT OF THE VIRTUAL MANIPULATOR

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Abstract. Future robotic systems will perform complex tasks in space. In these applications the dynamic coupling between manipulators and spacecraft can cause problems. A new and effective analytical method, called the Virtual Manipulator, is introduced for studying the kinematics and dynamics of manipulators in space. It is shown that the kinematics and dynamics of space manipulator systems can be described relatively easily using the Virtual Manipulator method. The approach may aid in the design and development of future manipulator systems in space.

Keywords. Manipulators in space; virtual manipulators; mobile manipulators; robotic manipulators; manipulator kinematics; manipulator dynamics.

INTRODUCTION

Robotic manipulators are potentially very useful for performing complex tasks in space (Akin, et. al., 1983; Meintel and Schappell, 1982). These tasks include the repair and servicing of satellites and the construction of space stations in orbit. Problems arise in these tasks due to the dynamic coupling between manipulators and the spacecraft which carry them. For example, link movements of a manipulator will disturb the attitude of its spacecraft. This in turn, will adversely affect the manipulator's precision, and reduce the on orbit life of the system by consuming excessive attitude control fuel. Such problems are not encountered in current industrial manipulators which generally are rigidly attached to their work places. Researchers working on the control of space manipulators have yet to deal with the dynamic coupling problem (Lee, Bekey, and Bejczy, 1985; Kohn and Healy, 1986).

This paper presents a new and effective analytical method for studying the dynamics of manipulators mounted on spacecraft, by introducing the concept of the Virtual Manipulator (VM). The VM is an ideal kinematic chain connecting a point fixed in inertial space, called the Virtual Ground (VG) to any point on the real manipulator. Motions of the system, i.e. the vehicle, manipulator and payload, are easily described in terms of the virtual system motions, a system whose base is fixed at a point in inertial space. This model is very effective in understanding and calculating kinematic and dynamic properties of space manipulator systems. For example, it can be used for inverse kinematic solutions, workspace analysis, dynamic analysis and dynamic path planning. It includes the manipulator-vehicle system interactions, and yields simpler solutions than conventional analysis methods. The VM also gives substantially increased understanding of the manipulator-vehicle dynamic interactions and can aid in the design and development of future manipulator systems for use in space.

A MODEL OF MANIPULATORS IN SPACE

Future space manipulator systems will consist of one or more manipulators carried by a vehicle, such as shown in Fig. 1. The vehicle will be capable of six degrees-of-freedom motion, and will have reaction jets for attitude and position control. The system's useful life will usually be limited by the amount of reaction jet fuel available to control the vehicle's position and orientation. The manipulator itself will probably be electrically powered by photovoltaic arrays and use no reaction jet fuel. A major consideration will be for the system to perform tasks with little or no use of the reaction jets.

In the development of the VM it is assumed that reaction jets are not used, and hence, the external forces/torques acting on the complete system are assumed to be small. Furthermore, the manipulator and vehicle are assumed to be composed of rigid bodies.

THE VIRTUAL MANIPULATOR

The Virtual Manipulator (VM) is a massless kinematic chain terminating at an arbitrary point on the real manipulator. Its base is the Virtual Ground (VG) which is an imaginary fixed point in inertial space. It is proven below that for a given system the properties of the VM and location of the VG are fixed.

VMs exist for many different manipulator structures, such as open or closed chains, single or many branched arms, revolute or prismatic joints. The discussion in this paper will be limited to manipulators composed of serial chains with revolute connections. Although VMs exist for any point on the real manipulator, this paper deals with VMs whose end points coincide with real manipulators' end effectors.

The VG is defined to be the center of mass of the manipulator-vehicle system. From elementry mechanics, when there are no external forces, such as from reaction jets, the VG will be fixed in inertial space. It will not move due to any internal forces of the system such as joint torques, or due to any manipulator motions.

Figure 2 shows a schematic drawing of an N body spatial manipulator system. The first body in the chain represents the vehicle which carries the manipulator. The N^{th} body is a combination of the payload and the last link. The i^{th} joint is called J_i, and C_i is the center of mass of the i^{th} body. The vector defining the axis of rotation of J_i is called $\mathbf{A}_i$, and the angle θ_i is the rotation of the i^{th} joint. The vectors $\mathbf{R}_i$ and $\mathbf{L}_i$ connect C_i to J_i and J_i to C_{i+1}, respectively. The vector $\mathbf{R}_N$ connects C_N to the end effector, E. The vectors $\mathbf{R}_i$ and $\mathbf{L}_{i-1}$ are fixed relative to the i^{th} link, and hence the angle between these vectors is constant for all system configurations.

The location of the VG for this system in inertial space, the center of mass of the system, can be found by knowing some initial position of the system. The vector $\mathbf{S}(0)$ defines the known location of E with respect to some inertial reference frame N. Then the location of the VG, the vector $\mathbf{T}$, can be obtained from:

$$\begin{aligned}\mathbf{T} &= [M_N\mathbf{S}(0) + M_{N-1}(\mathbf{S}(0) - \mathbf{L}_{N-1} - \mathbf{R}_{N-1}) + \dots + \\ &\quad + M_1(\mathbf{S}(0) - \mathbf{L}_{N-1} - \mathbf{R}_{N-1} - \dots - \mathbf{R}_1)]/M_{tot} \qquad (1)\end{aligned}$$

where: $M_{tot} = M_1 + M_2 + M_3 + \dots + M_N$.

The vector $\mathbf{T}$ is constant for all time. The vector $\mathbf{S}$ which locates the center of mass of the N^{th} body at any time, t, is given by:

$$\mathbf{S}(t) = \mathbf{T} + [(\mathbf{R}_1 + \mathbf{L}_1)M_1 + \dots + (\mathbf{R}_{N-1} + \mathbf{L}_{N-1})\sum_{q=1}^{N-1} M_q]/M_{tot} \qquad (2)$$

Equation (2) can be written in the form:

$$\mathbf{S}(t) = \mathbf{T} + (\mathbf{D}_1 + \mathbf{H}_1) + \dots + (\mathbf{D}_{N-1} + \mathbf{H}_{N-1}) \qquad (3)$$

where the vectors $\mathbf{D}_i$ and $\mathbf{H}_i$ are defined by:

$$\mathbf{D}_i = \mathbf{R}_i \sum_{q=1}^{i} M_q/M_{tot} \qquad (i = 1, 2, \dots, N) \qquad (4)$$

$$\mathbf{H}_i = \mathbf{L}_i \sum_{q=1}^{i} M_q/M_{tot} \qquad (i = 1, 2, \dots, N-1) \qquad (5)$$

Defining the vectors $\mathbf{V}_i$ as

$$\begin{aligned}\mathbf{V}_1 &= \mathbf{D}_1 \\ \mathbf{V}_i &= \mathbf{H}_{i-1} + \mathbf{D}_i \qquad (i = 2, 3, \dots, N) \qquad (6)\end{aligned}$$

it can be seen that the chain of vectors

$$\mathbf{V}_1 + \mathbf{V}_2 + \dots + \mathbf{V}_N,$$

represents a manipulator which has its base fixed at the VG and whose end point is at E. This is the Virtual Manipulator, see Fig. 3. The i^{th} VM link is $\mathbf{V}_i$. It begins at the $i-1^{th}$ VM joint and ends at the i^{th} VM joint. The last *N-1* VM joints are taken as revolute, because the real manipulator has revolute joints. The axes of the i^{th} VM joint and the i^{th} real manipulator joint are taken parallel in the initial manipulator position. The first VM joint, which connects the first virtual link to the VG, is taken as a spherical joint.

The VM joint movements depend on the real system movements. The rotations of the first VM joint, a spherical joint, are taken equal to the rotations of the spacecraft with respect to inertial space. The amount of rotation of the i^{th} virtual and real joints are always equal.

The VM constucted, as described above, has some very useful properties. They are:

As the system moves:

a The VM link lengths do not change.

b The axes of the i^{th} virtual and real joints always remain parallel.

c The VM end point always remain coincident with the manipulator end effector.

The proofs for these properties are contained in Appendix A. Property (c) can be extended to construct a VM with an end point on any desired point on the real manipulator (Vafa, 1987).

Table 1 contains the properties of a simple planar manipulator and its VM, which are both shown in Fig. 4. It should be noted that the VM method is not restricted to planar systems.

TABLE 1. Characteristics of a manipulator and its VM.

Body no	M (Kg)	R (m)	L (m)	D (m)	H (m)
1	50	1.0	1.0	0.33	0.33
2	50	0.75	0.75	0.5	0.5
3	50	0.5	-	0.5	-

VM APPLICATIONS

The VM has a number of applications. The motion of the free floating actual manipulator system can be described by motion of the much simpler VM, which has a fixed base in inertial space. It can be used, among other things, for inverse kinematic calculations, work space definition, path planning, and reduction of dynamic disturbances to the vehicle due to manipulator motions (Vafa, 1987; Vafa and Dubowsky, 1987). It can also be used to simplify the formulation of the system dynamic equations of motion. Here its use for this purpose is outlined.

An N link manipulator carried on a vehicle has N+6 degrees-of-freedom, six for vehicle position and orientation, and N for the manipulator joint variables. Using conventional analysis N+6 dynamic equations of motion must be written. This number can be reduced to N+3 by applying conservation of linear momentum, a cumbersome procedure. Using the VM approach the result yields the N+3 equations directly. First, it is noted that it is possible to construct VMs for any arbitrary point P on the real manipulator (Vafa, 1987). All of these VMs will have the same joint angles but different link lengths. The location of P in inertial space can be written as a linear function of the link lengths:

$$\mathbf{X}_p = F(\theta)\mathbf{D}_p \qquad (7)$$

where:

$\mathbf{X}_p$: 3 by 1 inertial position vector of P,

$F(\theta)$: 3 by N+1 transformation matrix,

θ : N+3 element vector of VM joint angles, θ_i,

$\mathbf{D}_p$: N+1 element vector of VM link lengths.

Equation (7) can be used in either Lagrangian or Newtonian formulations to give simply and directly a complete set of N+3 equations of motion for the system in terms of the N+3 VM joint angles.

As an example, the equations of motion for the system shown in Fig. 5, using VM and conventional formulation, are shown in Table 2. The VM's equations of motion are clearly simpler than the conventional formulations of equations of motion.

TABLE 2. Equations of Motion by Conventional and VM Formulations. ($m_1 = m_2 = 1$ kg, $L_1 = L_2 = 1$ m)

Equations of motion with conventional formulation
$\begin{bmatrix} 2 & 0 & -\sin(\theta_1) & -\sin(\theta_2) \\ 0 & 2 & \cos(\theta_1) & \cos(\theta_2) \\ -\sin(\theta_1) & \cos(\theta_1) & 1 & \cos(\theta_1-\theta_2) \\ -\sin(\theta_2) & \cos(\theta_2) & \cos(\theta_1-\theta_2) & 1 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -\dot{\theta}_1^2\cos(\theta_1)-\dot{\theta}_2^2\cos(\theta_2) \\ -\dot{\theta}_1^2\sin(\theta_1)-\dot{\theta}_2^2\sin(\theta_2) \\ \dot{\theta}_2^2\sin(\theta_1-\theta_2) \\ \dot{\theta}_1^2\sin(\theta_1-\theta_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \tau_1-\tau_2 \\ \tau_2 \end{bmatrix}$
Equations of motion with VM formulation
$\begin{bmatrix} 0.5 & 0.5\cos(\theta_1-\theta_2) \\ 0.5\cos(\theta_1-\theta_2) & 0.5 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0.5\dot{\theta}_2^2\sin(\theta_1-\theta_2) \\ 0.5\dot{\theta}_1^2\sin(\theta_1-\theta_2) \end{bmatrix} = \begin{bmatrix} \tau_1-\tau_2 \\ \tau_2 \end{bmatrix}$

SUMMARY AND CONCLUSION

In this paper, the concepts of Virtual Manipulators and Virtual Grounds were introduced. The construction and properties of Virtual Manipulators was presented, and one of its applications was discussed. This is a new concept and further research is required to demonstrate its full capabilities.

ACKNOWLEDGMENT

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APPENDIX A. VM PROPERTIES

It will be proven here that as the manipulator moves the VM does not change its properties. It is also proven that, as the system moves through a given set of joint angles, the VM with the same joint motions will keep its end point coincident with the real manipulator end effector, as shown in Fig. A1.

Imagine that the real manipulator goes through some movement. For the new manipulator position a new VM, VM*, can be constructed so that its end point coincides with the manipulator end effector. It is proven below that if the VM corresponding to the real manipulator's original position goes through the identical joint rotations as the real manipulator went through, it will coincide with the VM*.

For the new real manipulator position the vectors $\mathbf{R}_i^*$ and $\mathbf{L}_i^*$ are defined in the same manner as the vectors $\mathbf{R}_i$ and $\mathbf{L}_i$ which were defined earlier, and vectors $\mathbf{D}_i^*$ and $\mathbf{H}_i^*$ are found by application of equations (4) and (5). These are shown in Fig. A1. The i^{th} link of VM* is defined by $\mathbf{H}_{i-1}^* + \mathbf{D}_i^*$, see equation (6). The new real joint axes are labelled $\mathbf{A}_i^*$ and are parallel to the new virtual joint axes.

For a revolute joint system, the magnitudes R_i and L_i are constants. That is:

$$R_i^* = R_i$$
$$L_i^* = L_i$$

and since the mass of the system does not change, equations (4) and (5) show that the magnitudes:

$$D_i^* = D_i$$
$$H_i^* = H_i$$

The angle between $\mathbf{R}_i$ and $\mathbf{L}_{i-1}$ will not change as the i^{th} link moves, because the links are assumed to be rigid. This angle is equal to the angle between $\mathbf{D}_i$ and $\mathbf{H}_{i-1}$ and to the angle between $\mathbf{D}_i^*$ and $\mathbf{H}_{i-1}^*$. Therefore, the angle between $\mathbf{D}_i^*$ and $\mathbf{H}_{i-1}^*$ is equal to the angle between $\mathbf{D}_i$ and $\mathbf{H}_{i-1}$. Thus the vector $\mathbf{H}_{i-1} + \mathbf{D}_i$, which represents the i^{th} link of the original VM, and the vector $\mathbf{H}_{i-1}^* + \mathbf{D}_i^*$, which represents the i^{th} link of VM*, are equal. Therefore, links of the original VM and the VM* will have the same size.

Now it will be proven that when the original VM goes through the same joint rotations, θ, as the original real manipulator went through, it will coincide with VM*. This proves that as the real manipulator moves through θ in joint space, the end point of the VM moving through θ in its joint space will always coincide with the end effector of the real manipulator.

First assume the VM joint rotations are equal to the joint rotations of the real manipulator and the rotations of the first VM joint are equal to the vehicle rotations with respect to inertial space. In this case, the VM link $\mathbf{D}_1$ will coincide, after the movement, with the link $\mathbf{D}_1^*$ of VM*, and since the joint axes of rotation are fixed relative to the links, axis of rotation $\mathbf{A}_1$ of the VM will coincide, after movement, with the axis $\mathbf{A}_1^*$ of VM* .

Rotations about $\mathbf{A}_1$ of the second link of the real manipulator, represented by $\mathbf{L}_1$ and $\mathbf{R}_2$, and the second link of the VM, represented by $\mathbf{H}_1$ and $\mathbf{D}_2$, are assumed equal. Furthermore, the axis $\mathbf{A}_1$, after movement is the same as and hence parallel to A_i^*. Therefore, after movement, the vectors H_1 and D_2 will be parallel to L_i^* and R_i^*, respectively. From equations (4) and (5), L_i^* and R_i^* are also parallel to H_i^* and D_2^*, respectively. Thus, after movement, the vectors H_1 and D_2 are parallel to H_i^* and D_2^*. Since the second link of the original VM after movement, has the same size as the second link of VM*, D_1 will be coincident with D_i^* and the vectors representing them are parallel, it follows that the second link of the original VM will, after movement, coincide with the second link of VM*.

By extension it follows that, when the original VM goes through joint rotations corresponding to those of the real manipulator, its links will coincide with those of the VM that corresponds to the new position of the real manipulator, i.e. VM*. This proves that the VM moving along the same path in joint space as the real manipulator is sufficient to describe motion of the real manipulator end effector .

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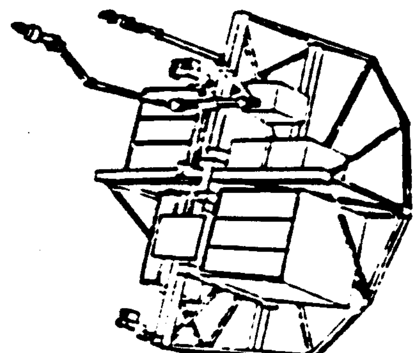

Fig. 1. A Space Manipulator System.

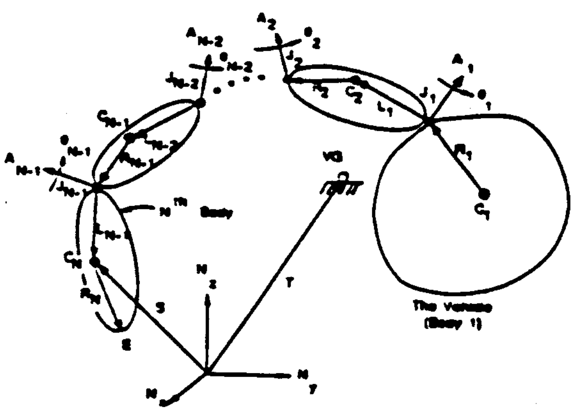

Fig. 2. N Body Revolute System in Space.

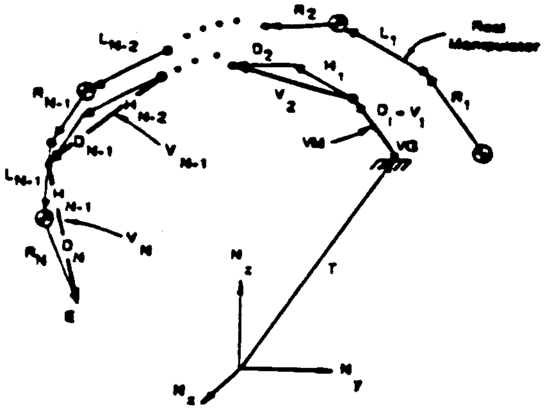

Fig. 3. N Body System and its VM.

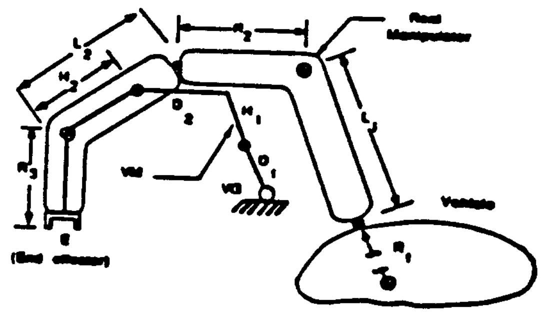

Fig. 4. A Three Body Planar System and its VM.

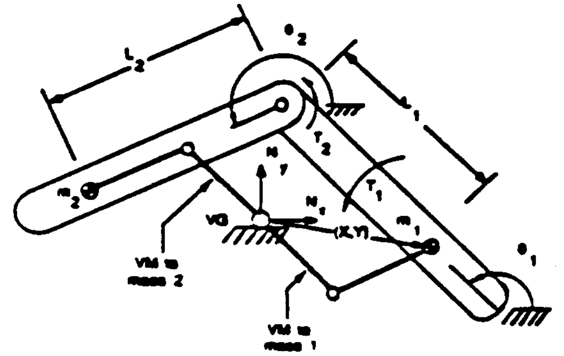

Fig. 5. A Planar Two Body System and the VMs for Center of Mass of Each Body.

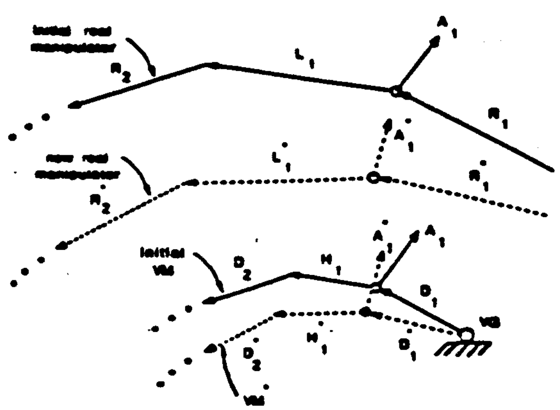

Fig. A1. A Manipulator System and its VM for Two Configurations.